ITP–UH–15/15

Supermembrane limit of Yang-Mills theory

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Abstract

We consider Yang-Mills theory with $N{=}1$ super translation group in eleven auxiliary dimensions as the structure group. The gauge theory is defined on a direct product manifold $\Sigma_3 \times S^1$, where Σ_3 is a three-dimensional Lorentzian manifold and S^1 is a circle. We show that in the infrared limit, when the metric on S^1 is scaled down, the Yang-Mills action supplemented by a Wess-Zumino-type term reduces to the action of an M2-brane.

1. Introduction and summary. The theory of membranes and supermembranes has been developed for a long time [1]-[9].[1] Supermembranes are basic objects (M2-branes) of M-theory, which are needed for constructing an effective theory of multi-M2-branes [9]. In this paper we show that the action of supermembranes moving in d=11 flat N=1 extended superspace can be obtained from a Yang-Mills action functional on $\Sigma_3 \times S^1$ amended by a Wess-Zumino-type term when S^1 shrinks to a point.

Our construction is based on the adiabatic approach to differential equations (introducing "slow" and "fast" variables) which for a direct product manifold[2] $Z = X \times Y$ is equivalent to the introduction of a metric $g_X + \varepsilon^2 g_Y$ with a real parameter $\varepsilon \in [0, \infty)$ and a consideration of the limit $\varepsilon \to 0$ [10, 11].[3] The adiabatic limit method has been applied to the description of the scattering of monopoles (i.e. constructing time-dependent solutions of the Yang-Mills-Higgs model), and it has been shown that in the limit $\varepsilon \to 0$ the scattering of monopoles is parametrized by geodesic motion on the moduli space $\mathcal{M}_n$ of n-monopoles [15, 16]. In other words, the Yang-Mills-Higgs system on $\mathbb{R}^{3,1} = \mathbb{R}^{0,1} \times \mathbb{R}^{3,0}$ for "slow time" reduce to a sigma model on $\mathbb{R}^{0,1}$ (time axis) with $\mathcal{M}_n$ as the target space.

In four dimensions, when dim$Z$=4, one has dim$X$=1, 2 or 3 and dim$Y$=3, 2 or 1, respectively. In [10] the adiabatic method was applied to the Yang-Mills instanton equations on a direct product $X \times Y$ of two Riemann surfaces, and it was shown that instanton solutions on $X \times Y$ are in a *one-to-one correspondence* with holomorphic maps from X into the moduli space $\mathcal{M}$ of *flat connections* on Y. In this case the Yang-Mills action reduces to the action of a sigma model on X while Y shrinks to a point. The sigma-model target space is $\mathcal{M}$, and holomorphic maps $X \to \mathcal{M}$ are the sigma-model instantons. The same result for the Lorentzian signature with $X = \mathbb{R}^{1,1}$ and $Y = T^2$ (two-torus) was derived in [12]: Yang-Mills theory on $\mathbb{R}^{1,1} \times T^2$ in the infrared limit $\varepsilon \to 0$ (the size of T^2 tends to zero) reduces to a sigma model on $\mathbb{R}^{1,1}$ whose target space is the moduli space of flat connections on T^2. In [13, 14] the same approach was applied to Yang-Mills theory[4] on $\mathbb{R}^{2,1} \times S^1$. It was shown that Yang-Mills theory on $\mathbb{R}^{2,1} \times S^1$ reduces to a sigma model on $\mathbb{R}^{2,1}$ whose target space is the space of vacua that arise in the compactification on S^1. Finally, the adiabatic approach is natural and especially helpful in studying Yang-Mills instantons in more than four dimensions as it was shown in [11, 17] (see also [18] and references therein).

To sum up, Yang-Mills theory on a manifold $X \times Y$ with metric $g_X + \varepsilon^2 g_Y$ flows in the infrared limit $\varepsilon \to 0$ to a sigma model on X whose target space is the moduli space $\mathcal{M}$ of flat connections on Y when dim$Y \le 2$. In our short paper we reverse this logic. For a given sigma model on X we construct a Yang-Mills model on $X \times Y$ such that in the infrared limit $\varepsilon \to 0$ one gets back the initial sigma model. In [19, 20] this algorithm was carried out for the bosonic string and for the Green-Schwarz superstring in a d=10 Minkowski background. Here we apply this idea to the sigma model describing a supermembrane in a d=11 Minkowski background [5, 7] and introduce a Yang-Mills model on $\Sigma_3 \times S^1$ whose low-energy limit recovers the supermembrane action on Σ_3.

[1] See [8, 9] for historical reviews and more references.

[2] The direct product structure is not necessary for the application of the adiabatic method. In general, it is enough if there is a fibration $Z \to X$ or if X is a calibrated submanifold of Z.

[3] In the physics literature this limit is called infrared or low-energy limit (see e.g. [12, 13, 14]).

[4] In fact, in [12]-[14] the authors considered $\mathcal{N}$=4 and $\mathcal{N}$=2 super-Yang-Mills theories but the restriction to the pure Yang-Mills subsector does not change the picture.

2. Lie supergroup G. We consider Yang-Mills theory on a direct product manifold $M^4 = \Sigma_3 \times S^1$, where Σ_3 is a three-dimensional Lorentzian manifold with local coordinates x^a, $a, b, \ldots = 0, 1, 2$, and a metric tensor $g_{\Sigma_3} = (g_{ab})$, and on the circle S^1 of unit radius parametrized by $x^3 \in [0, 2\pi]$ we choose the metric $g_{S^1} = (g_{33})$ with $g_{33} = 1$. Then $(x^\mu) = (x^a, x^3)$ are local coordinates on M^4 with the metric tensor $(g_{\mu\nu}) = (g_{ab}, g_{33}), \mu, \nu, \ldots = 0, \ldots, 3$. Having in mind open membranes, we assume that Σ_3 has a Lorentzian boundary $\Sigma_2 = \partial\Sigma_3$. For closed membranes, Σ_2 is the empty set.

As the Yang-Mills structure group on M^4 we consider the coset $G = \mathrm{SUSY}(N{=}1)/\mathrm{SO}(10{,}1)$ (cf. [7]), where SUSY(N=1) is the super Poincaré group in d=11 dimensions. The coset G is the super translation group in d=11 auxiliary dimensions. Its generators span the Lie superalgebra $\mathfrak{g} = \mathrm{Lie}\, G$,

$$\{\xi_A, \xi_B\} = (\gamma^\alpha C)_{AB}\xi_\alpha \ , \quad [\xi_\alpha, \xi_A] = 0 \ , \quad [\xi_\alpha, \xi_\beta] = 0 \ , \tag{1}$$

where γ^α are the gamma matrices in d=11, C is the charge conjugation matrix, $\alpha = 0, \ldots, 10$ and $A = 1, \ldots, 32$. The coordinates on G are denoted by X^α and by the components θ^A of a Majorana spinor $\theta = (\theta^A)$, whose conjugate is $\bar\theta = \theta^\top C$. The one-forms

$$\Pi^\Delta \ = \ \{\Pi^\alpha, \Pi^A\} \ = \ \{\mathrm{d}X^\alpha - \mathrm{i}\,\bar\theta\,\gamma^\alpha\theta \ , \ \mathrm{d}\theta^A\} \tag{2}$$

form a basis of (left-invariant) one-forms on G [5, 7]. On the superalgebra $\mathfrak{g} = \mathrm{Lie}\, G$ we introduce the scalar product $\langle\cdot\cdot\rangle$ such that

$$\langle\xi_\alpha\, \xi_\beta\rangle = \eta_{\alpha\beta} \ , \quad \langle\xi_\alpha\, \xi_A\rangle = 0 \quad \text{and} \quad \langle\xi_A\, \xi_B\rangle = 0 \ , \tag{3}$$

where $(\eta_{\alpha\beta}) = \mathrm{diag}(-1, 1, \ldots, 1)$ is the Lorentzian metric on $\mathbb{R}^{10,1}$.

3. Action functional. Let us consider the gauge potential $\mathcal{A} = \mathcal{A}_\mu \mathrm{d}x^\mu$ with values in $\mathfrak{g}$ and the $\mathfrak{g}$-valued gauge field

$$\mathcal{F} \ = \ \tfrac{1}{2}\mathcal{F}_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \qquad \text{with} \qquad \mathcal{F}_{\mu\nu} \ = \ \partial_\mu\mathcal{A}_\nu - \partial_\nu\mathcal{A}_\mu + [\mathcal{A}_\mu, \mathcal{A}_\nu] \ , \tag{4}$$

where $[\,\cdot\,,\,\cdot\,]$ is the commutator or anti-commutator depending on the Grassmann parity of its arguments. On $\Sigma_3 \times S^1$ we have the obvious splitting

$$\mathrm{d}s^2 \ = \ g_{\mu\nu}\mathrm{d}x^\mu\mathrm{d}x^\nu \ = \ g_{ab}\mathrm{d}x^a\mathrm{d}x^b + (\mathrm{d}x^3)^2 \ , \tag{5}$$

$$\mathcal{A} \ = \ \mathcal{A}_\mu\mathrm{d}x^\mu \ = \ \mathcal{A}_a\mathrm{d}x^a + \mathcal{A}_3\mathrm{d}x^3 \ , \tag{6}$$

$$\mathcal{F} \ = \ \tfrac{1}{2}\mathcal{F}_{\mu\nu}\mathrm{d}x^\mu \wedge \mathrm{d}x^\nu \ = \ \tfrac{1}{2}\mathcal{F}_{ab}\mathrm{d}x^a \wedge \mathrm{d}x^b + \mathcal{F}_{a3}\mathrm{d}x^a \wedge \mathrm{d}x^3 \ . \tag{7}$$

On $M^4 = \Sigma_3 \times S^1$, with its boundary $\partial M^4 = \partial\Sigma_3 \times S^1 = \Sigma_2 \times S^1$, the (super)group of gauge transformations is naturally defined as (see e.g. [21, 22])

$$\mathcal{G} \ = \ \{g : M^4 \to G \mid g_{|\partial M^4} = \mathrm{Id}\} \ . \tag{8}$$

This corresponds to a framing of the gauge bundle over the boundary. For closed membranes we keep the framing over S^1.

Employing the adiabatic approach [10, 11, 15, 16, 22, 23], we deform the metric (5),

$$\mathrm{d}s^2_\varepsilon \ = \ g^\varepsilon_{\mu\nu}\,\mathrm{d}x^\mu\mathrm{d}x^\nu \ = \ g_{ab}\,\mathrm{d}x^a\mathrm{d}x^b + \varepsilon^2(\mathrm{d}x^3)^2 \ , \tag{9}$$

where $\varepsilon \in [0,\infty)$ is a real parameter. This is equivalent to scaling the radius of our circle, replacing it with S^1_ε of radius ε. Indices are raised by $g^{\mu\nu}_\varepsilon$, and we have

$$\mathcal{F}^{ab}_\varepsilon \;=\; g^{ac}_\varepsilon g^{bd}_\varepsilon \mathcal{F}_{cd} \;=\; \mathcal{F}^{ab} \qquad \text{and} \qquad \mathcal{F}^{a3}_\varepsilon \;=\; g^{ac}_\varepsilon g^{33}_\varepsilon \mathcal{F}_{c3} \;=\; \varepsilon^{-2}\mathcal{F}^{a3} \ , \tag{10}$$

where indices in $\mathcal{F}^{\mu\nu}$ have been raised by the non-deformed metric tensor components $g^{\mu\nu}$. In addition we have $\det(g^\varepsilon_{\mu\nu}) = \varepsilon \det(g_{\mu\nu})$.

We consider the Yang-Mills action functional with a cosmological constant Λ of the form

$$S_\varepsilon \;=\; \int_{M^4} \mathrm{d}^4x \, \sqrt{|\det g_{\Sigma_3}|}\, \left\{ \frac{\varepsilon^2}{2} \langle \mathcal{F}_{ab}\, \mathcal{F}^{ab}\rangle + \langle \mathcal{F}_{a3}\, \mathcal{F}^{a3}\rangle + \Lambda \right\} \ . \tag{11}$$

For $\varepsilon = 1$ and $\Lambda = 0$ it coincides with the standard Yang-Mills action. The value of Λ will be fixed later.

4. Euler-Lagrange equations. For the deformed metric the Yang-Mills equations take the form

$$\varepsilon^2 D_a \mathcal{F}^{ab} + D_3 \mathcal{F}^{3b} \;=\; 0 \tag{12}$$

$$\text{and} \qquad D_a \mathcal{F}^{a3} \;=\; 0 \ . \tag{13}$$

Allowing also the metric g_{Σ_3} on Σ_3 to vary, its Euler-Lagrange equations give the energy-momentum constraint

$$T^\varepsilon_{ab} \;=\; \varepsilon^2 \big(g^{cd} \langle \mathcal{F}_{ac}\, \mathcal{F}_{bd}\rangle - \tfrac14\, g_{ab} \langle \mathcal{F}_{cd}\, \mathcal{F}^{cd}\rangle \big) + \langle \mathcal{F}_{a3}\, \mathcal{F}_{b3}\rangle - \tfrac12\, g_{ab} \big(\langle \mathcal{F}_{c3}\, \mathcal{F}^{c3}\rangle + \Lambda \big) \;=\; 0 \ . \tag{14}$$

In the adiabatic limit $\varepsilon \to 0$, our equations (12)-(14) become

$$D_3 \mathcal{F}^{3b} \;\equiv\; \partial_3 \mathcal{F}^{3b} + [\mathcal{A}_3, \mathcal{F}^{3b}] \;=\; 0 \ , \tag{15}$$

$$D_a \mathcal{F}^{a3} \;\equiv\; \sqrt{|\det g_{\Sigma_3}|}^{\,-1}\, \partial_a \big(\sqrt{|\det g_{\Sigma_3}|}\, g^{ab} \mathcal{F}_{b3} \big) + [\mathcal{A}_a, \mathcal{F}^{a3}] \;=\; 0 \ , \tag{16}$$

$$T^0_{ab} \;\equiv\; \langle \mathcal{F}_{a3}\, \mathcal{F}_{b3}\rangle - \tfrac12 g_{ab} \big(\langle \mathcal{F}_{c3}\, \mathcal{F}^{c3}\rangle + \Lambda \big) \;=\; 0 \ . \tag{17}$$

5. Moduli space. Let us recall how one considers the reduction of Yang-Mills theory from $\mathbb{R}^3 \times S^1_\varepsilon$ to $\mathbb{R}^3$ while S^1_ε shrinks to a point for an ordinary compact Lie group G [13, 14].[5] Firstly, one keeps in the lagrangian (11) only the zero modes $\mathcal{A}^0_3$ in the Fourier expansion on S^1_ε, which are nothing but the Wilson lines, whose moduli are parametrized by coordinates ϕ^α of the maximal torus in G. These moduli produce a term $\mathcal{F}_{a3}\, \mathcal{F}^{a3} = \delta_{\alpha\beta}\, \partial_a \phi^\alpha \partial^a \phi^\beta$ in the lagrangian. Secondly, for $\mathcal{F}_{ab}$ smoothly depending on ε, the first term in the lagrangian (11) vanishes. However, it was observed [13, 14] that for Dirac monopoles the components $\mathcal{F}_{ab}$ are related with the magnetic photon, having only one component $\tilde{\mathcal{A}}^0_3$ along S^1_ε, via

$$\varepsilon_{abc} \mathcal{F}^{bc} \;=\; \varepsilon^{-1}\, \partial_a \tilde{\mathcal{A}}^0_3 \ , \tag{18}$$

where the ε^{-1} appears from the metric dependence of the Hodge star operator. These monopole configurations correspond to 't Hooft lines around the circle S^1_ε. They survive in the limit $\varepsilon \to 0$,

[5] For simplicity, we restrict ourselves to the pure Yang-Mills subsector of the supersymmetric theories in [13, 14].

yielding in the lagrangian (11) an additional term proportional to $\delta_{\alpha\beta}\,\partial_a\psi^\alpha\partial^a\psi^\beta$, where ψ^α are coordinates on the Cartan torus in the dual group $G^\vee$.

In our case the situation is different since our supermembrane moves in a noncompact superspace, namely $G = \mathrm{SUSY}(N{=}1)/\mathrm{SO}(10,1)$. For any fixed $x^a \in \Sigma_3$, a generic framed $\mathcal{A}_3$ is parametrized by the moduli space

$$\Omega G \;=\; \mathrm{Map}(S^1_\varepsilon, G)/G \;=\; LG/G\ , \tag{19}$$

i.e. the based loop group, and it can be written in the form

$$\mathcal{A}_3 \;=\; \hat h^{-1}\partial_3 \hat h \;=\; h^{-1}\mathcal{A}_3^0 h + h^{-1}\partial_3 h \qquad \text{with} \qquad \hat h = h_0 h \in \Omega G \quad \text{and} \quad \mathcal{A}_3^0 = h_0^{-1}\partial_3 h_0 \in \mathfrak{g}\ , \tag{20}$$

where $h\in\Omega G$ and $h_0 \in G \subset \Omega G$. Note that neither $\hat h$ nor h belong to the gauge group. In fact, (20) defines a map $\hat h \mapsto h_0$ from ΩG to G. The Wilson lines $\mathcal{A}_3^0$ are parametrized by G. Since our aim is the supermembrane moving in G, we choose the magnetic photon component $\tilde{\mathcal{A}}_3^0$ to vanish. Furthermore, in the spirit of the adiabatic approach it is assumed that all moduli of $\mathcal{A}_3$ are functions of $x^a \in \Sigma_3$, i.e. both functions h and h_0 depend on x^a via their moduli. We denote by $\mathcal{N}$ the space of all $\mathcal{A}_3$ given by (20), and we define the projection $\pi : \mathcal{N} \to G$ since we want to keep only $\mathcal{A}_3^0$ in the limit $\varepsilon \to 0$.

6. Effective action. The variable $\mathcal{A}_3^0$, as introduced in (20), depends on $x^a\in\Sigma_3$ only via the moduli parameters $(X^\alpha, \theta^A) \in G$. Then the moduli of $\mathcal{A}_3^0$ define a map

$$(X,\theta):\ \Sigma_3 \to G \qquad \text{with} \qquad \big(X(x^a), \theta(x^a)\big) = \big(X^\alpha(x^a), \theta^A(x^a)\big)\ . \tag{21}$$

The map (21) is not arbitrary, it is constrained by the equations (15)-(17). The derivative $\partial_a\mathcal{A}_3$ belongs to the tangent space $T_{\mathcal{A}_3}\mathcal{N}$. With the help of the projection $\pi : \mathcal{N} \to G$ with fibres Q, one can decompose $\partial_a\mathcal{A}_3$ into two parts,

$$T_{\mathcal{A}_3}\mathcal{N} \;=\; \pi^* T_{\mathcal{A}_3^0} G \oplus T_{\mathcal{A}_3} Q \qquad \Leftrightarrow \qquad \partial_a \mathcal{A}_3 \;=\; \Pi_a^\Delta \xi_{\Delta 3} + D_3\epsilon_a\ , \tag{22}$$

where $\Delta = (\alpha, A)$ and

$$\Pi_a^\alpha \;=\; \partial_a X^\alpha - \mathrm{i}\,\bar\theta\gamma^\alpha\partial_a\theta \qquad \text{and} \qquad \Pi_a^A \;=\; \partial_a\theta^A\ . \tag{23}$$

In (22), ϵ_a are $\mathfrak{g}$-valued parameters ($D_3\epsilon_a \in T_{\mathcal{A}_3}Q$), and the vector fields $\xi_{\Delta 3}$ on G can be identified with the generators $\xi_\Delta = (\xi_\alpha, \xi_A)$ of G.

On $\xi_{\Delta 3}$ we impose the gauge-fixing condition

$$D_3\xi_{\Delta 3} = 0 \quad \overset{(22)}{\Longrightarrow} \quad D_3 D_3 \epsilon_a = D_3\partial_a\mathcal{A}_3\ . \tag{24}$$

Recall that $\mathcal{A}_3$ is determined by (20) and $\mathcal{A}_a$ are yet free. In the adiabatic approach one can naturally choose $\mathcal{A}_a = \epsilon_a$ (cf. [15, 23]), where ϵ_a are defined from (24). Then one obtains

$$\mathcal{F}_{a3} \;=\; \partial_a\mathcal{A}_3 - D_3\mathcal{A}_a \;=\; \partial_a\mathcal{A}_3 - D_3\epsilon_a \;=\; \Pi_a^\Delta\xi_{\Delta 3}\ \in T_{\mathcal{A}_3^0}G\ . \tag{25}$$

Substituting (25) into (15), we see that the latter is resolved due to (24). Plugging (25) into the action (11) with $\varepsilon \to 0$ and fixing $\Lambda = -1$, we obtain the effective action

$$S_0 \;=\; 2\pi \int_{\Sigma_3} \mathrm{d}^3x\ \sqrt{|\det g_{\Sigma_3}|}\,\Big(g^{ab}\,\Pi_a^\alpha\,\Pi_b^\beta\,\eta_{\alpha\beta} - 1\Big)\ . \tag{26}$$

It coincides with the kinetic part of the supermembrane action [5]. One may also show (cf. [19]) that the equations (16) are equivalent to the Euler-Lagrange equations for (X^α, θ^A) following from (26). Finally, substituting (25) into (17), we arrive at

$$\Pi_a^\alpha \, \Pi_b^\beta \, \eta_{\alpha\beta} - \tfrac{1}{2}\, g_{ab} \left(g^{cd}\, \Pi_c^\alpha \, \Pi_d^\beta \, \eta_{\alpha\beta} - 1 \right) \;=\; 0 \tag{27}$$

which may also be obtained from (26) by varying the metric.

From (27) it follows that

$$g_{ab} \;=\; \eta_{\alpha\beta}\, \Pi_a^\alpha \, \Pi_b^\beta \;, \tag{28}$$

and, after putting this back into (26), we get the standard Nambu-Goto lagrangian for the supermembrane. It is obvious that for $\theta = 0$ the bosonic membrane action remains.

7. Wess-Zumino-type term. The action (26) is not the full supermembrane action, since the latter needs also a Wess-Zumino-type term [5, 7]. Continuing our 'reverse engineering' strategy, we look for an addition to the Yang-Mills action (11) which in the infrared limit $\varepsilon \to 0$ will give us this Wess-Zumino-type term. This addition can be incorporated as follows. We extend Σ_3 to a Lorentzian 4-manifold Σ_4 with boundary $\Sigma_3 = \partial\Sigma_4$ and (local) coordinates $x^{\hat a}$, $\hat a = 0, 1, 2, 4$. On Σ_4 one introduces the four-form [5, 7]

$$\Omega_4 \;=\; \langle \Pi \wedge \Pi \wedge \Pi \wedge \Pi \rangle \;=\; f_{\Delta\Lambda\Sigma\Gamma}\, \Pi^\Delta \wedge \Pi^\Lambda \wedge \Pi^\Sigma \wedge \Pi^\Gamma \;=\; \hat{\mathrm{d}}\bar\theta \gamma_{[\alpha}\gamma_{\beta]} \wedge \hat{\mathrm{d}}\theta \wedge \Pi^\alpha \wedge \Pi^\beta \;=\; \hat{\mathrm{d}}\Omega_3 \tag{29}$$

for $\Pi := \Pi_{\hat a} \mathrm{d}x^{\hat a} = \Pi_{\hat a}^\Delta \mathrm{d}x^{\hat a} \xi_\Delta$, where $\hat{\mathrm{d}} = \mathrm{d}x^{\hat a}\partial_{\hat a}$. The explicit form of the constants $f_{\Delta\Lambda\Sigma\Gamma}$ and the three-form Ω_3 can be found in [5, 7]. Then one adds to the action (26) the term

$$S_{WZ} \;=\; \int_{\Sigma_4} \Omega_4 \;=\; \int_{\Sigma_3} \Omega_3 \;, \tag{30}$$

which completes the M2-brane action. In the set-up we investigate here, we take the direct product manifold $\Sigma_4 \times S^1$, extend the index a in (23) to $\hat a = 0, 1, 2, 4$ and introduce one-forms on Σ_4,

$$F_3 \;:=\; \mathcal{F}_{\hat a 3} \mathrm{d}x^{\hat a} \;. \tag{31}$$

Adding (with a proper coefficient) the Wess-Zumino-type term

$$S_{WZ}^{YM} = \int_{\Sigma_4 \times S^1} f_{\Delta\Lambda\Sigma\Gamma}\, F_3^\Delta \wedge F_3^\Lambda \wedge F_3^\Sigma \wedge F_3^\Gamma \wedge \mathrm{d}x^3 \tag{32}$$

to the action functional S_ε from (11) with $\Lambda = -1$, we obtain the gauge-field action which in the adiabatic limit $\varepsilon \to 0$ becomes the M2-brane action. This implies that features of Yang-Mills theory with the action (11)+(32) for $\varepsilon \neq 0$ can be reduced to properties of supermembranes by taking the limit $\varepsilon \to 0$.

Acknowledgements

This work was partially supported by the Deutsche Forschungsgemeinschaft grant LE 838/13.

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